

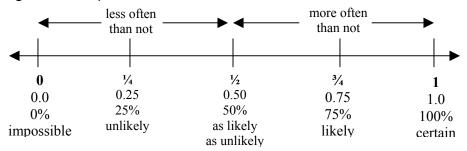
PROBABILITY Session 5

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Probability	Probability Background Information	155			
	Between 0 and 1	158	3.23, 4.19, 5.17	Recording Sheets, Probability Statements	Scissors
	Lay It on the Line	161	4.19, 5.17	Lay It On The Line Statements	
	What's In the Bag?	163	2.24, 3.23, 4.19, 5.17	What's In The Bag?	Paper bags, color tiles
	Fair or Not Fair	165	3.23, 4.19	Fair or Not? Game Sheet	Dice
	The Regatta	167	4.19, 5.17, 6.20, 7.14	Regatta Game Board, Log, Variations	Number cubes, 12 counters per participant, markers
	Tree Diagrams	178	6.20	Tree Diagrams	
	The Real Meal Deal	180	5.17, 6.20, 7.15	Real Meal Menu	Chart paper
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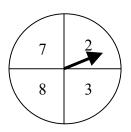


Probability Background Information

A measure used to express the likelihood of an event happening is considered **probability**. Probability can give you information about the likelihood of an event happening, but it is never a guarantee. The probability of an event happening can be expressed as a number from 0 to 1.



The probability of an event happening is the ratio of the number of positive outcomes to the number of possible outcomes. For example, suppose one wanted to know the probability of spinning a prime number on the spinner below. One could calculate the probability as follows:



P (prime number) =
$$\frac{3 \text{ prime numbers}}{4 \text{ numbers total}} = \frac{3}{4}$$

Ways to express probability:

There is a 3 out of 4 chance of this event occurring.

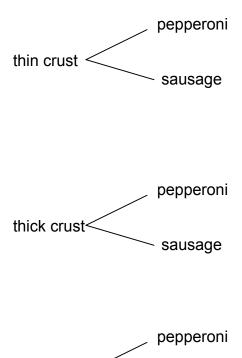
When conducting a probability experiment, data about the outcomes is being collected. This is called *sampling*. Sampling may not match the expected probability, but if conducted over extended periods of time, it should come fairly close. It is important to discuss with children the idea that what actually happens (experimental) doesn't always fit with what is expected to happen (theoretical).

In order to calculate the probability of an event, one may wish to create a sample space. A **sample space** lists all the possible outcomes of an event. For the above spinner, our sample space would be 2, 3, 7, 8. These are all of the possible outcomes of spinning the spinner.



Another way to determine the outcomes in a sample space is to draw a *tree diagram*.

Example: A pizza shop offers three styles of crust and two different toppings. How many different combinations of crusts and toppings are there?



sausage

deep dish <

There are 6 possible combinations. The probability of each combination occurring is 1/6. Therefore, the probability of a customer ordering a deep-dish pepperoni compared to all others is about 17%.

Simulation can be used to understand natural fluctuations or variation in data. For instance, if we were attempting to determine whether a spinner with 6 spaces is "fair", we might use simulation. A fair spinner would produce each outcome (1-6) an equal number of times over many, many spins. For instance, if we spun the spinner 60 times, we would expect each outcome to occur approximately 10 times. Notice we say approximately because we expect some variation – for instance, maybe eight 1s, thirteen 2s, etc. To determine if the spinner is fair, we need to know how much variation to expect—if we only got three 1s, is the spinner unfair or, in other words, is this variation too big to believe that the spinner is fair? By simulating outcomes that occur when we



make a selection randomly, we can better understand what is normal variation and what is abnormal variation.



Activity: Between 0 and 1

Format: Individual or small group

Objectives: Participants will classify statements as impossible, likely or

certain to help establish the concept of probability.

Related SOL: 3.23, 4.19, 5.17

Materials: Recording sheets, probability statements, scissors, glue or

tape, Between 0 and 1 Activity Sheet and Probability

Statements

Time Required: 20 minutes

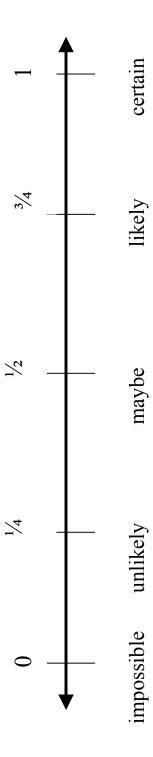
Directions:

1. Briefly discuss the vocabulary with the participants to gauge their understanding of the terms, impossible, likely, and certain.

- 2. Display the recording sheet on the overhead and discuss the number line concept.
- 3. Ask the participants, individually or in small groups, to assign values to the vocabulary words discussed. Record them on the transparency as participants record on their copy of the sheet.
- 4. Distribute the Probability Statements Activity Sheet. Have the participants cut the statements apart. Each statement should be placed on the number line recording sheet in the position corresponding to the likelihood of the event occurring.
- 5. When all of the number lines have been completed, ask groups members or individuals to share the results and discuss the similarities and differences.



Between 0 and 1





Between 0 and 1

Probability Statements

It will rain tomorrow.	You will have homework tonight.
Pizza will be served for lunch.	Your school has a principal.
The sun will rise tomorrow.	You will go to bed before 9:00 tonight.
You will have two birthdays this year.	You will go to Disney World sometime.
Your teacher is over 18 years old.	You will get tails when you flip a coin.
Two students will be absent tomorrow.	You throw a 4 on a die.
You will ride in a bus before the end of the school year.	Your teacher will let you have extra recess.
It will take you more than 1 hour to do your homework.	On your way to school you will see a live dinosaur.



Activity: Lay It on the Line

Format: Individual or small group

Objectives: Participants will classify statements as impossible or certain to

help establish the concept of probability.

Related SOL: 4.19, 5.17

Materials: Lay It on the Line Statements (1 per participant or group), glue,

scissors

Time Required: 20 minutes

<u>Background</u>: Referencing a number line should help participants understand

that the likelihood of an event occurring ranges from 0 to 1. It should also help them in evaluating the reasonableness of their calculations (an unlikely event should not have a probability of

0.90).

Directions:

- 1. Briefly discuss the vocabulary with participants to gauge their understanding of the terms. Display the "Between 0 and 1" overhead from the previous activity and discuss the number line concept with them.
- 2. Ask the participants, individually or in small groups, to assign values to these other probability words: probable, impossible, certain, maybe, always, unexpected, unlikely, possible, even chance, and improbable. Participants should record them on the recording sheet.
- 3. Have the students cut the statements apart. Each statement should be placed on the Lay It on the Line Recording Sheet in the position corresponding to the likelihood of the event occurring. When all of the number lines have been completed, post them and discuss the similarities and differences.



Lay It on the Line Statements

It will rain tomorrow.	Your teacher is more than 16 years old.
You will be given a homework assignment in math sometimes this year.	You will ride on a jet plane before the end of the year.
Drop a rock in water and it will sink.	You will have two birthdays this year.
Your school has a principal.	You will go to bed before 8:00 tonight.
Trees will sing in the afternoon.	You walk into the yard and see a live dinosaur.
You will learn to play the flute.	You will toss a die and show a 6.
The sun will rise tomorrow morning.	You will toss a coin and show a head.
You will go to Disneyland sometime.	Two students will be absent tomorrow.



<u>Activity</u>: What's In the Bag?

Format: Pairs

Objectives: Participants will conduct simple probability experiments to

predict outcomes.

Related SOL: 2.24, 3.23, 4.19, 5.17

Materials: Paper bags, color tiles, recording sheet

Time Required: 20 minutes

Directions:

1. Organize participants into pairs.

2. Give each pair a paper bag with 10 color tiles inside (7 blue and 3 red).

- Pairs will pull out one tile (without looking into the bag) and record the color on their recording sheet. The tile should be returned to the bag. Each pair will pull out tiles following this process a total of ten times
- 4. As pairs finish, have them record their results on a class graph at the front of the room.
- 5. When the class data is complete, have pairs look at the total number of blue and red tiles pulled and then make their prediction about the number of blue and red tiles in the bag.
- 6. After everyone has had a chance to predict, discuss the predictions and reasons why.
- 7. Have pairs look into their bags and record the actual results.
- 8. Discuss why their predictions may have differed from the actual number. What was helpful in making their predictions?
- 9. Repeat using bags with four colors of tiles. Discuss differences noted.



What's In the Bag?

Pick one tile from the paper bag. Record the color on the table below. Put the tile back into the bag. Choose another tile. Repeat this process 9 more times.

Blue									
Red									
My predic	tion:				Act	tual res	ults:		
There are		bl	ue tiles	S.	blu	e tiles _.			_
There are		re	ed tiles		rec	l tiles _			
Let's try a	ıgain wi	ith four	colors	!					
Blue									
Red									
Yellow	,								
Green									
My predic	tion:	·				Act	tual res	sults:	
There are	!	bl	ue tiles	3 .	blu	e tiles _.			_
There are red tiles.		rec	l tiles _						
There are	!	y	ellow ti	iles.	yel	low tile	s		
There are	!	g	reen til	les.	gre	en tiles	3		



Activity: Fair or Not Fair?

Format: Pairs

Objectives: Participants will be able to determine sample space and

fairness of a game.

Related SOL: 3.23, 4.19

Materials: 1 die per pair, Fair or Not Fair? Game/Recording Sheet (1 per

pair)

Time Required: 20 minutes

Background: The sample space of an experiment is nothing more than the

collection of all the possible outcomes for that experiment. In this case, all the possible results of the roll of a die are 1, 2, 3, 4, 5, 6. These are all of the possible outcomes of rolling a die – the sample space. A game is considered fair if the likelihood of

winning is the same as the likelihood of losing.

Directions:

1. Ask participants to predict the winner of each of the games described in the sheets distributed.

- 2. Have the participants pair up. (Ham and Cheese, Please! could be used for pairing.)
- 3. Each game should be played 20 times and the result of each game recorded.
- 4. Did the results match the prediction?
- 5. Was the game fair? Why or why not?
- 6. What could be done to make the game fair if it were not?
- 7. Have the participants construct the sample space to help in making the decision of how to make the game fair.



Fair or Not Fair? Game Sheet

Game Number	Player	What wins
1	Α	the numbers 1, 2, or 3
	В	the numbers 4, 5, or 6
2	Α	any odd number
	В	any even number
3	Α	any number less than 4
	В	any number greater than 4
		a 4 does not win
4	Α	any prime number
	В	any composite number

Fair or Not Fair? Recording Sheet

Game Number	Winner	Tally	Total
1	A: 1, 2, 3		
	B: 4, 5, 6		
2	A: odd numbers		
	B: even numbers		
3	A: numbers < 4		
	B: numbers >4		
4	A: prime numbers		
	B: composite numbers		



Activity: The Regatta

Format: Pairs or small group

Objectives: Participants will conduct simple probability experiments to

predict outcomes using a game format and two number cubes. Participants will investigate probability, sample space and tree

diagrams.

Related SOL: 4.19, 5.17, 6.20, 7.14

<u>Materials</u> Two number cubes per pair, The Regatta game board (1 per

pair), The Regatta transparency, 12 beans or small counters per pair, The Log Book Recording Sheet, large graph paper,

colored pencils or markers

Time Required: 40 minutes

<u>Background</u>: Participants will be conducting an experiment – any activity that

has two or more clearly discernible results or outcomes. As a result of the experiment, participants should be able to list the

sample space for that experiment. Sample space is a collection of all possible outcomes. Based on this sample space, participants should be able to determine the probability of an event occurring. Probability is defined as the ratio of the

number of favorable outcomes to all outcomes of an

experiment. An event is defined as any subset of the outcomes or any subset of the sample space – usually the outcome we

are looking for, a favorable outcome.

When all possible outcomes of a simple experiment are equally likely (each result is as likely to occur as every other), the theoretical probability of an event is

number of outcomes in the event number of possible outcomes

Experimental probability is based on the results of an experiment rather than a theoretical analysis of the experiment. Theoretical probability is based on a logical analysis of an experiment, not on experimental results.

The probability of an event is always 0, 1, or any number between 0 and 1. An impossible event has a probability of 0.



A certain event has a probability of 1.

Probabilities between 0 and 1 may be expressed as a ratio, a decimal, or a percent.

Directions:

Part I.

- 1. Explain to the participants that they will play a game using two number cubes or dice. Place The Regatta game board transparency on the overhead and place 12 counters on the starting line, one to represent each yacht.
- 2. Explain how the yachts move across the course. After a player rolls the cubes, he/she moves the yacht whose number is the sum of cubes ahead ONE space. For example, if a three and a five are rolled, the player moves yacht #8 one space forward.
- 3. Participants should predict which yacht would reach the finish line first. Record several responses. Have all participants predict, by a show of hands, which yacht they think will win.
- 4. Demonstrate the game on the overhead. Select two participants to model taking turns rolling the cubes and moving the appropriate yacht.
- 5. Pause frequently during the demonstration to look at the results. Ask participants if they notice any patterns in the way the yachts are moving. Ask participants to comment on why some of the yachts have not moved yet. Participants may want to predict again as the race progresses, changing their favorite as they watch the race. As new information is received, refinements in predictions are allowed.
- 6. Post the class graph and record the winning yacht.
- 7. Remind participants about accurate recording of their results.
- 8. Distribute The Regatta game boards, counters, and number cubes.
- 9. Have the participants begin playing The Regatta. Each race should be recorded on the class graph as soon as it is completed.
- Participants should play the game at least four times. All results should be recorded on the class graph so that a large amount of data can be collected for analysis in Part II.
- 11. Provide time for discussion of The Regatta game. Ask for participants to discuss any surprises they found in the results. Ask for "true statements" about the graph.



Part II.

- 1. Review the results displayed on the class graph. Ask participants to explain the results shown.
- 2. Lead the group in a discussion of the mathematics involved in The Regatta. Ask, "How many ways could you make a sum of six with two dice?" Remind them that this can include reversals, such as 4 + 2 and 2 + 4. For example, a green four and a white two is a different arrangement of the dice than a white four and a green two. On the overhead, record all the possible ways to make six using two dice.

Die 1	Die 2
1	5
2	4
3	3
4	2
5	1

- 3. List the ways to make several other numbers such as the number that won the most, or numbers that won few races.
- 4. On the overhead, show The Log Recording Sheet.
- 5. To help participants understand The Log Recording Sheet, color the dice on the transparency. For example, if participants used a red die and a green die, shade the dice in the horizontal row red and the dice in the vertical row green. Demonstrate how to fill in part of the chart by finding the sum and placing the sum in the appropriate box.
- Distribute The Log Recording Sheet to participants. Have them color the dice on The Log to match the ones they used when they played The Regatta. Have them complete The Log.
- 7. Check to see that participants have completed The Log correctly. Display a completed Log on the overhead.
- 8. Ask the participants to discuss any patterns they notice on The Log. Ask if The Log shows all of the ways to make sum six, as well as other sums. Ask the participants to discuss the frequency of obtaining each sum. How does this frequency relate to the winners of The Regatta?
- 9. Ask the participants how many ways a seven can be made. From a review of The Log, they should determine that there are six ways to make a seven. How many possible sums can be obtained by rolling two dice? From The Log, there are 36 possible outcomes when two dice are rolled.



10. Have the participants use the chart to show numerical ways to represent probabilities. One way of representing the probability of each number 1 through 12 is shown below:

```
a sum of 7:
                  6 out of 36
                                or 1 out of 6
                                                   or 1/6
a sum of 6 or 8: 5 out of 36
                                or 1 out of 7
                                                      1/5
                                                   or 1/9
a sum of 5 or 9: 4 out of 36
                               or 1 out of 9
a sum of 4 or 10: 3 out of 36
                               or 1 out of 12
                                                  or 1/12
a sum of 3 or 11: 2 out of 36
                                or 1 out of 18
                                                  or 1/18
a sum of 2 or 12: 1 out of 36
                                or 1/36
a sum of 1:
                 0 out of 36
                                or 0
```

These fractional probabilities can be used to remind participants of the relationship of a written number to the real world.

Part III.

1. Give each pair a calculator and have them calculate the experimental probability of each winner in The Regatta. Then have them calculate the theoretical probability of each sum listed on The Log. A comparison of the two probabilities should be made.

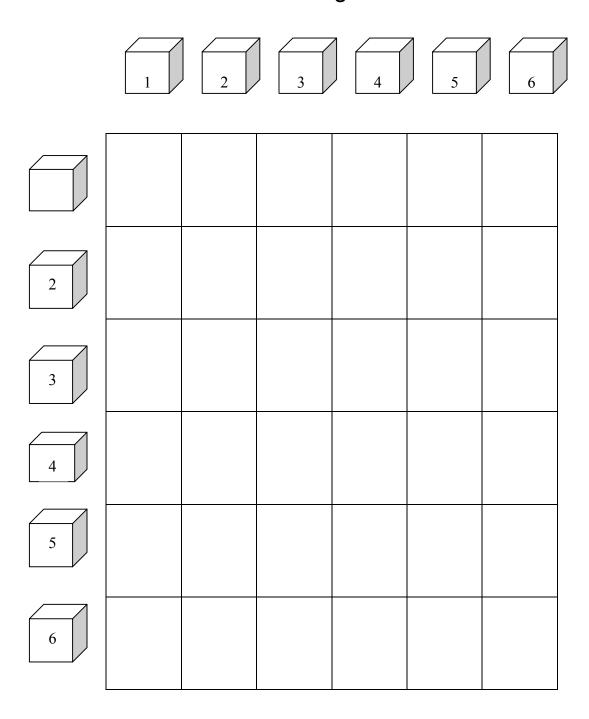


The Regatta Game Board

1		Ů,	1
2			2
3		Ĵ	3
4		J.	4
5		Ĵ.	5
6		L	6
7		Ĵ	7
8		Ţ	8
9		Ĵ	9
10		Ĵ.	10
11		Ĵ.	11
12			12



The Log





Variation for Part II.

- 1. Lead the group into the development of the *sample space* all possible outcomes. Record all the sums previously identified on the chalkboard or overhead.
- 2 3 4 5 6 7 8 9 10 11 12
- 2. Ask "How many ways can I get a sum of 2?" ... "of 3?", and so on. Continue until all sum combinations are found. Record each response.

At this point, ask the participants if they see a pattern? If so, predict how many combinations are there to get a sum of 7? Record the predictions. Then, verify the number of combinations.

Ask "Can you predict how many combinations to get a sum of 8?" Record the prediction. Verify the number of combinations.

Participants may suggest (1,7); that's okay, remind them that number cubes have only 1 to 6 to work with.

Ask "What happened to our prediction? What went wrong?"



Do you want to predict how many combinations there are for 9? Record any predictions. Verify the combinations.

Continue in a similar fashion until the entire sample space is listed.

How many possible outcomes are there?

NOTE: This is a good time to construct a tree diagram and show participants how to determine the sample space with a tree diagram.

Now, which sum do you think should occur most often? Does this match your prediction, or the results shown on our graph? Why or why not?



Variation for Part I.

Ask the participants to predict the winner of each of the following games.

Rule of play:

Two dice are rolled; the sum of the faces of the two dice is computed.

Game	Player	What wins:
Number		
1	Α	Even sums
	В	Odd sums
2	Α	Sums of 2, 3, 4, 5, 6
	В	Sums of 8, 9, 10, 11, 12
3	Α	Sums that are prime
	В	Sums that are composite
4	Α	Sums > 7
	В	Sums < 7
5	Α	Sums of 4, 5, 6, 7
	В	Sums of 2, 3, 10, 11, 12

Pair the participants. Have each pair pick a game to play. Be sure all games are played by at least one pair. Have the pairs roll 50 times; one player rolls 25 times, while the other tallies and records the winner. After 25 rolls, have the players switch.

After the tallies are complete, combine all the data into one class chart and discuss the results.

Possible questions:

Do you think the game you played was fair or not? If it was fair, why? If it was not fair, why?

What could be done to make the game fair?



Variation for Part I: Two Dice Difference

How to Play: The game is played by 2 participants, each of whom has one sixsided die. Participants roll simultaneously and find the absolute (or positive) value of the difference of the numbers.

A wins if the value is 0, 1, or 2.

B wins if the value is 3, 4, or 5.

Pairs roll the dice 10 times and keep a record of wins and losses.

Collect class date.

Is the game fair? Why or why not?

Have the participants construct the sample space and calculate the theoretical probability of each outcome.

Based on the sample space, how could the game be made fair?

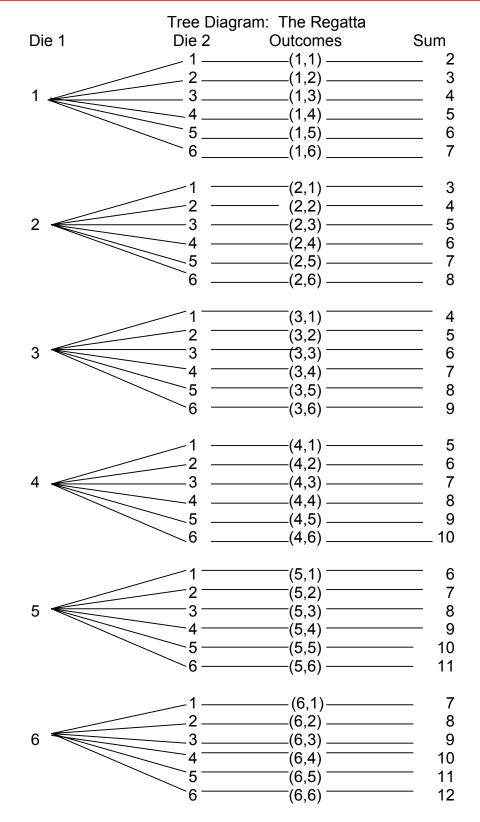
Two Dice Sum: Fair Game Recording Sheet

Use the line plot to tally the sums that were rolled.



Game Number	Winner	Tally	Total
1	A: even sums		
'	B: odd sums		
2	A: sum of 2, 3, 4, 5, 6		
2	B: sum of 8, 9, 10, 11, 12		
3	A: prime sum		
3	B: composite sum		
4	A: sum >7		
4	B: sum <7		
5	A: sum of 4, 5, 6, 7		
3	B: sum of 2, 3, 10, 11, 12		







Activity: Tree Diagrams

Format: Pairs or individually

Objectives: Participants will construct a tree diagram.

Related SOL: 5.17, 6.20

Materials: Tree Diagrams Recording Sheet

Time Required: 20 minutes

Directions:

1. Choose three participants to come to the front of the room. Try to choose people who are wearing different types of outfits.

- 2. Construct a tree diagram (as a total group) of all the possible combinations of outfits that can be made from the clothes the participants are wearing. For example: blue shirt (person 1), jeans (person 2), tennis shoes (person 3).
- 3. Continue with the Tree Diagrams Activity Sheet on constructing tree diagrams from pizza choices.
- 4. Discuss how the sample space changes when you add additional choices.





Tree Diagrams

You are trying to decide which pizza to order for dinner. Your choices for crust are: regular, thin, and deep dish. You only want one topping and will either choose pepperoni or sausage.

Construct a tree diagram to show the possibilities you have from which to choose one crust with one topping.

How would your sample space change if you added bacon as a third topping choice?



Activity: The Real Meal Deal

Format: Pairs

Objective: Participants will use a tree diagram and the Fundamental

Counting Principle to determine the sample space of an event.

Related SOL: 5.17, 6.20, 7.15

Materials: Real Meal Restaurant Menu, chart paper for tree diagrams

Time Required: 20 minutes

Background: The Fundamental Counting Principle is a method for finding the

number of ways that two or more events can occur by

multiplying the number of ways that each event can occur. The Principle states that, if successive choices are made, then the total number of choices is the product of the number of choices

at each stage.

For example, if you have 3 shirts and 2 pairs of jeans, then you have a total 6 different outfits to wear. Each shirt may be worn with each pair of jeans. There are 3 shirts times 2 pairs of

jeans for a total of 6 outfits.

A tree diagram is a visual way to see all of the outcomes.



Directions:

- 1. Based on the menu of the Real Meal Restaurant, participants will use the Fundamental Counting Principle to determine the number of different meals that can be served.
- 2. Based on customer wishes, participants will determine and display the choices using a tree diagram.



NOTE: This may be a good time to have participants go back to The Regatta and develop a tree diagram for the sample space formed by finding the sum of two dice.



REAL MEAL RESTAURANT

SANDWICHES: Ham and Turkey Club Hamburger

Rachael on Rye Deli Cold Cut Special

Sliced BBQ Pork BLT

FRENCH FRIES: small medium large

SALADS: Garden Salad <u>DRESSINGS</u>: Ranch

Chef Salad French

Cobb Salad Creamy Italian

BEVERAGES:

Soft Drinks: small medium large

Coke Pepsi Sprite

Tea: medium large Coffee: medium large

Milk: regular low-fat

- 1. How many possible meals can be served at the Real Meal, choosing only one item from each category?
- 2. How many choices are there if a customer wants the following:
 - a. a soft drink, sandwich, and fries? Display the choices with a tree diagram.
 - b. a sandwich, fries, and milk? Display the choices with a tree diagram.
 - c. salad with dressing and tea? Display the choices with a tree diagram.
 - d. a sandwich, salad with dressing, and coffee? Display the choices with a tree diagram.



Activity: Reflections on the Course

Format: Pairs, Large Group

Objective: Participants will discuss what they have learned and how they

plan to use what they have learned in their teaching.

Participants will make a commitment to revising their instruction

of Probability and Statistics, where appropriate.

Related SOL: All Probability and Statistics SOL

Materials: Paper and pencil

Time Required: 20 minutes

Directions:

- 1. Have participants work in pairs to discuss what they will do differently when teaching probability and statistics.
- 2. In a round-robin session, have participants share their future plans for teaching probability and statistics.